Lecture 22
Mutual Inductance & Transformers

Flux Linkages and Faraday's Law

Magnetic flux passing through a surface area A:

$$\phi = \int_A \mathbf{B} \cdot d\mathbf{A}$$

For a constant magnetic flux density perpendicular to the surface:

$$= BA$$

The flux linking a coil with N turns:

$$\lambda = N\phi$$

Faraday's Law

Faraday's law of magnetic induction:

$$e = \frac{d\lambda}{dt}$$

The voltage induced in a coil whenever its flux linkages are changing. Changes occur from:

- Magnetic field changing in time
- Coil moving relative to magnetic field

Lenz's Law

Lenz's law states that the polarity of the induced voltage is such that the voltage would produce a current (through an external resistance) that opposes the original change in flux linkages.

Mutual Inductance & Transformers

1. Determine the inductance and mutual inductance of coils given their physical parameters.
2. Understand ideal transformers and solve circuits that include transformers.
3. Use the equivalent circuits of real transformers to determine their regulations and power efficiencies.

Inductance and Mutual Inductance

Definition of inductance L:

$$L = \frac{Flux\,linkages}{current} = \frac{\lambda}{i}$$

Substitute for the flux linkages using $\lambda = N\phi$

$$L = \frac{N\phi}{i}$$

Inductance and Mutual Inductance

Substituting $\phi = \dfrac{Ni}{\mathcal{R}}$

$$L = \frac{N^2}{\mathcal{R}}$$

Faraday's Law

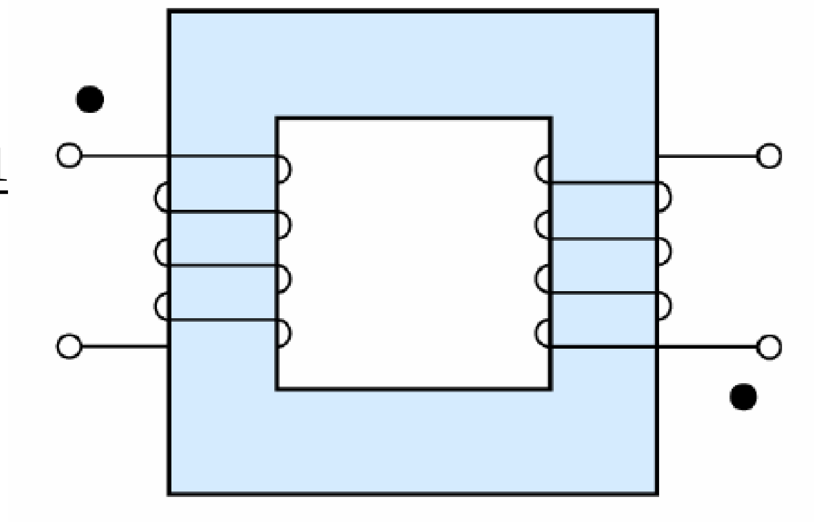
Voltage is induced in a coil when its flux linkages change:

$$e = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt}$$

Mutual Inductance

Self inductance for coil 1

$$L_1 = \frac{\lambda_{1\leftarrow 1}}{i_1} = \frac{\lambda_{11}}{i_1}$$



Self inductance for coil 2

$$L_2 = \frac{\lambda_{2\leftarrow 2}}{i_2} = \frac{\lambda_{22}}{i_2}$$

Mutual inductance between coils 1 and 2:

$$M = \frac{\lambda_{2\leftarrow 1}}{i_1} = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{1\leftarrow 2}}{i_2} = \frac{\lambda_{12}}{i_2}$$

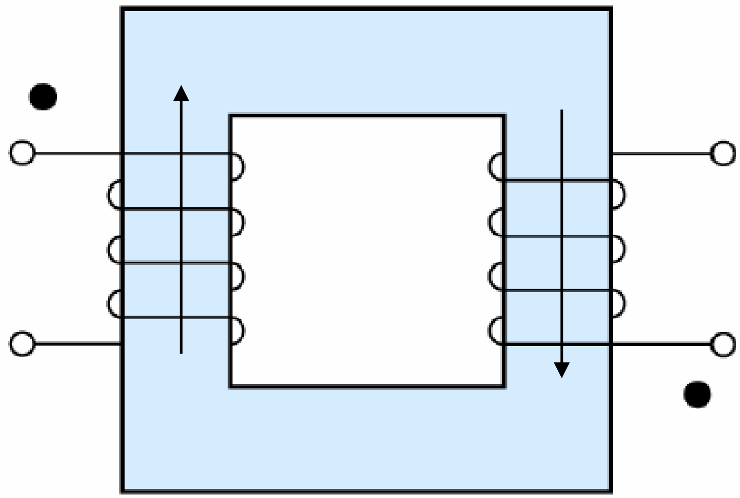
Mutual Inductance

Total fluxes linking the coils:

$$\lambda_1 = \lambda_{11} \pm \lambda_{12}$$

$$\lambda_2 = \lambda_{22} \pm \lambda_{21}$$

Mutual Inductance



Currents entering the dotted terminals produce aiding fluxes

Circuit Equations for Mutual Inductance

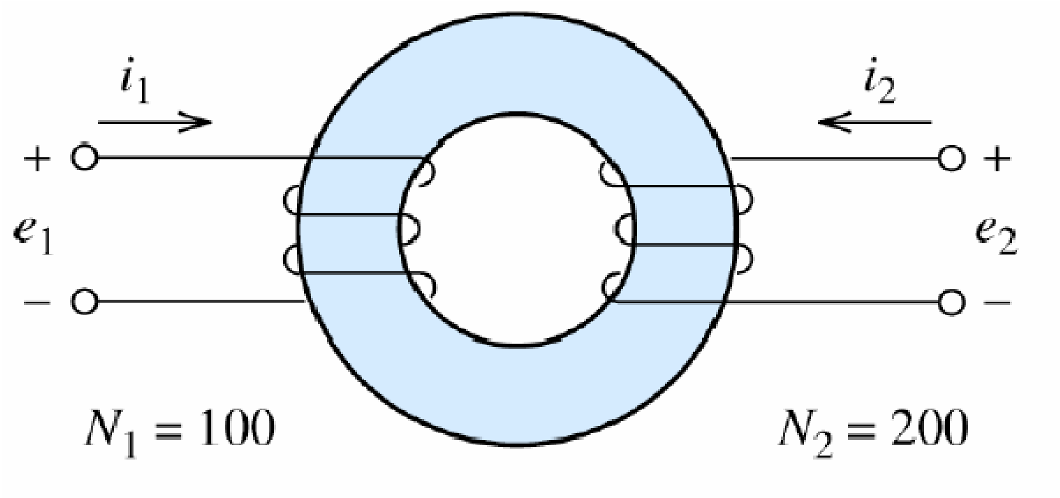
$$\lambda_1 = L_1 i_1 \pm M i_2$$

$$\lambda_2 = \pm M i_1 + L_2 i_2$$

$$e_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$e_2 = \frac{d\lambda_2}{dt} = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Example

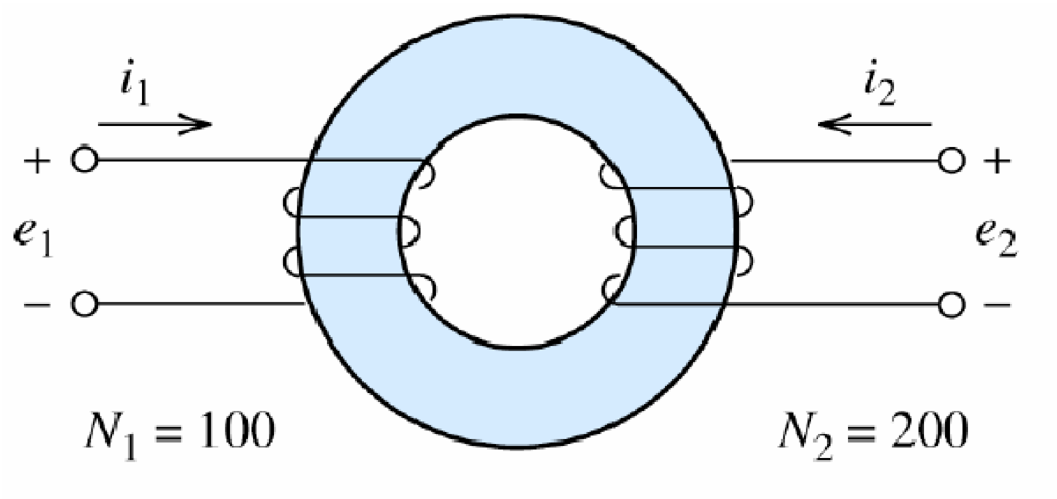


$$R = 10^7 \ (ampere - turns)/Weber$$

Self inductance: $$L_1 = \frac{N_1^2}{\mathcal{R}} = \frac{100^2}{10^7} = 1mH$$

$$L_2 = \frac{N_2^2}{\mathcal{R}} = \frac{200^2}{10^7} = 4mH$$

Example

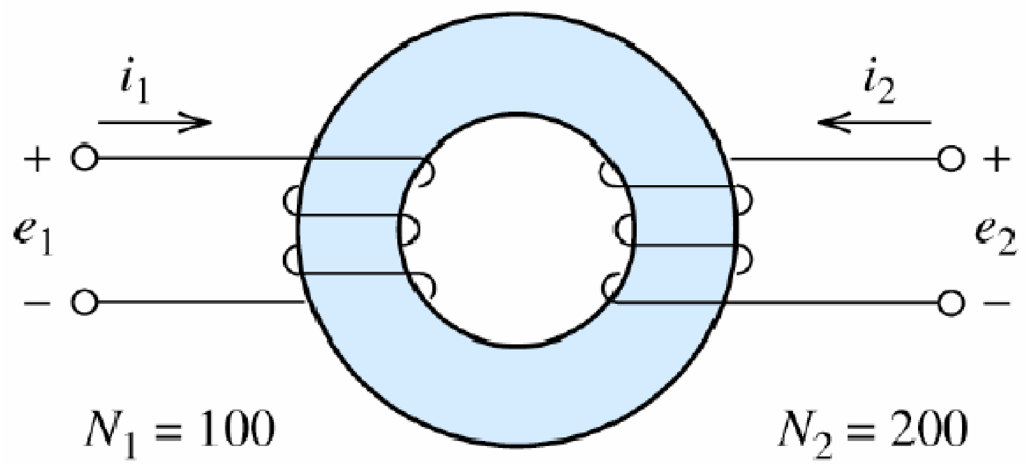


$$\phi_1 = \frac{N_1 i_1}{\mathcal{R}} = \frac{100 i_1}{10^7} = 10^{-5} i_1$$

$$\lambda_{21} = N_2 \phi_1 = 200 \times 10^{-5} i_1$$

Mutual inductance: $M = \dfrac{\lambda_{21}}{i_1} = 2mH$

Example

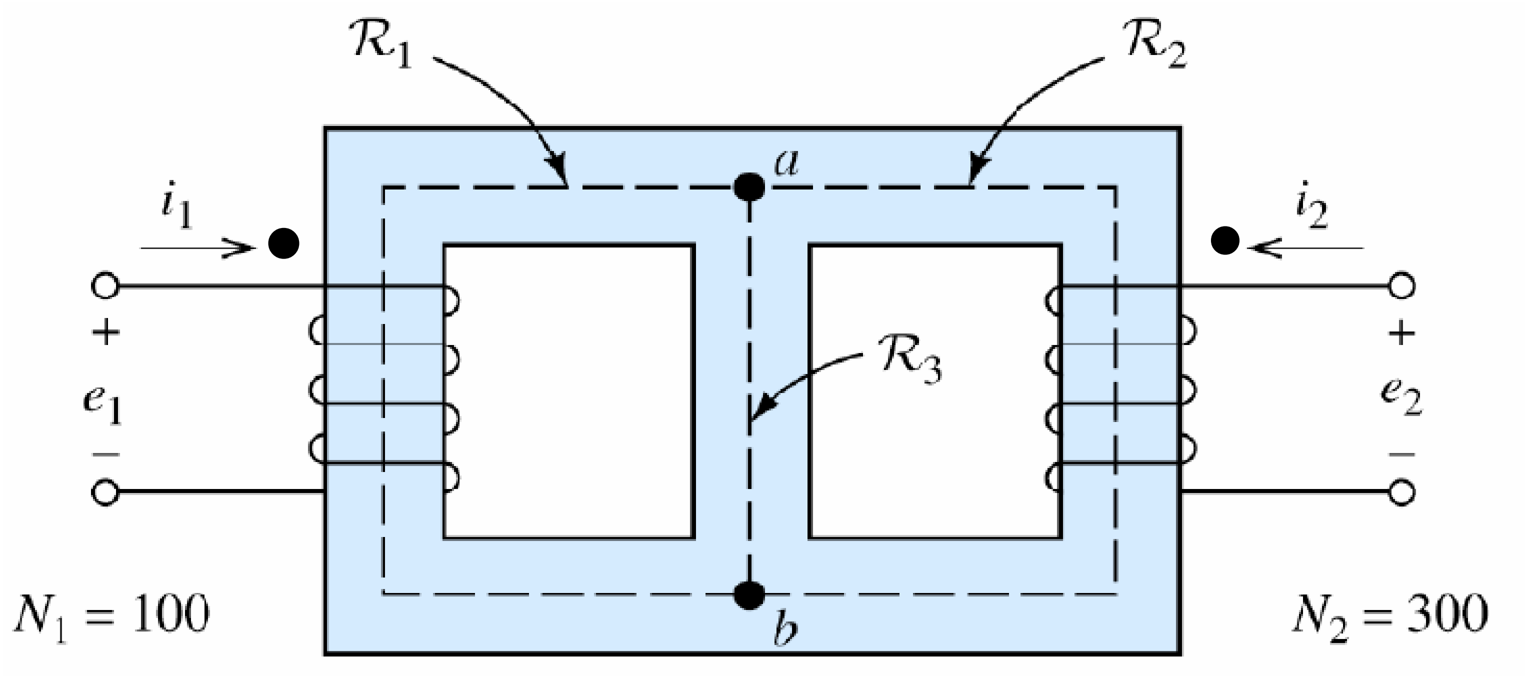


Does the flux produced by i_2 aid or oppose the flux produced by i_1?

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$e_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

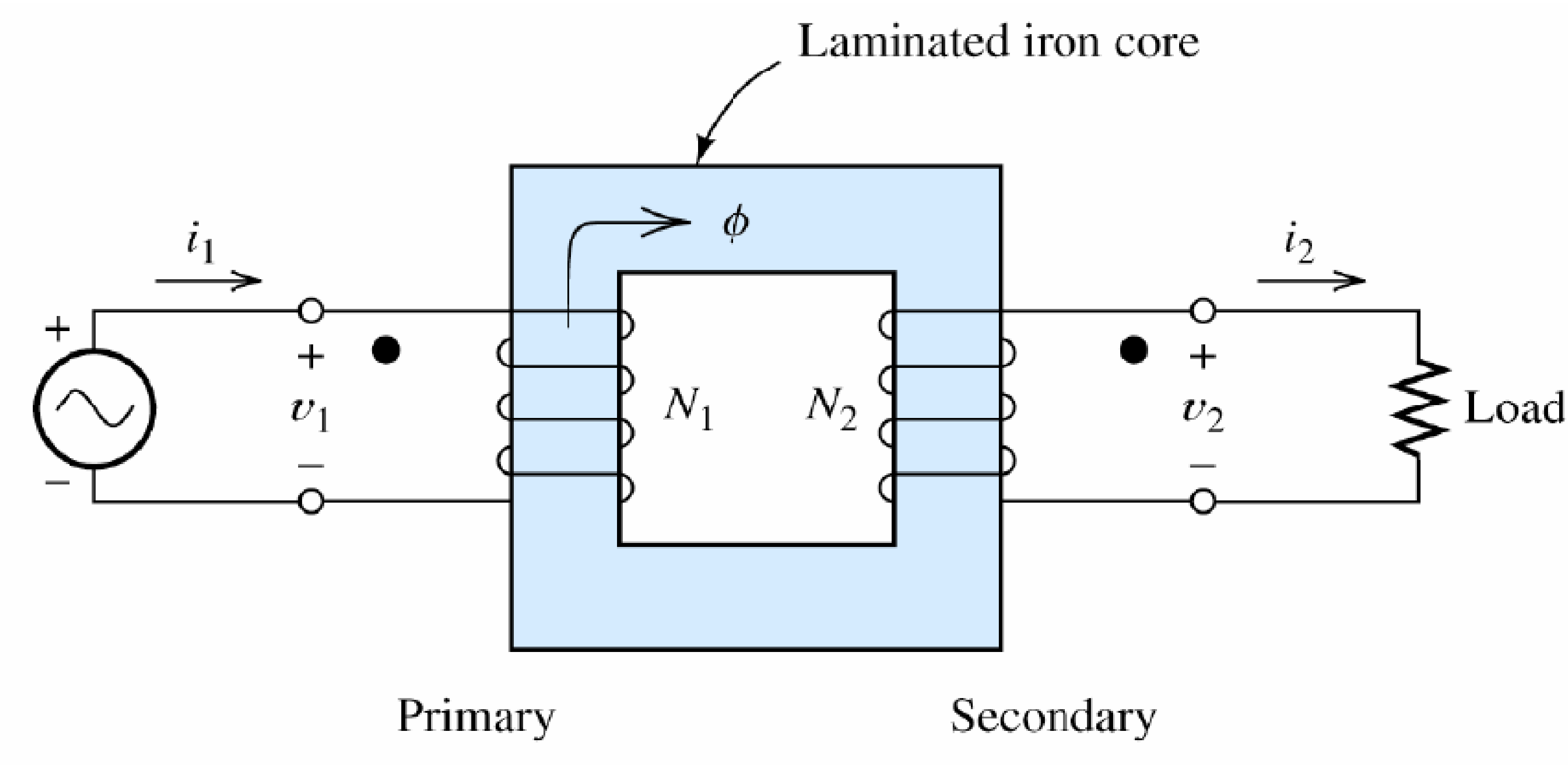
Exercise 15.13



Mark the dot for coil 2

Fluxes from 1 and 2 aid each other in paths 1&2, oppose in path 3

Transformers



Can be used to *step up* or *step down* ac voltages

Electrical Power Distribution

$$P_{delivered} = V_{rms} I_{rms} \cos(\theta)$$

$$P_{loss} = R_{line} I_{rms}^2$$

Minimize loss by increasing voltage and decreasing current. Modern transmission grids use ac voltages up to 765,000 Volts

War of the Currents

In the "War of Currents" era in the late 1880s, Nikola Tesla and Thomas Edison became adversaries due to Edison's promotion of direct current (DC) for electric power distribution over the more efficient alternating current (AC) advocated by Tesla.

During the initial years of electricity distribution, Edison's direct current was the standard for the United States and Edison was not disposed to lose all his patent royalties. Direct current worked well for the incandescent lamps that were the principal load of the day. From his work with rotary magnetic fields, Tesla devised a system for generation, transmission, and use of AC power. He partnered with George Westinghouse to commercialize this system.

War of the Currents

Edison's propaganda

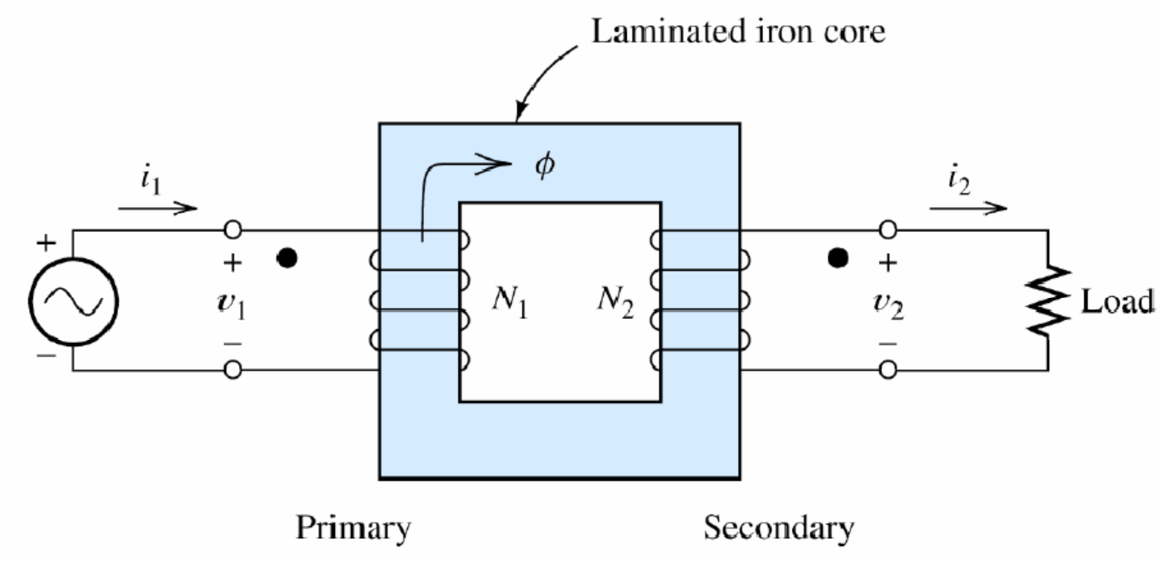
Edison went on to carry out a campaign to discourage the use of alternating current. Edison personally presided over several executions of animals, primarily stray cats and dogs, to demonstrate to the press that his system of direct current was safer than that of alternating current. Edison's series of animal executions peaked with the electrocution of Topsy the Elephant. He also tried to popularize the term for being electrocuted as being "Westinghoused".

War of the Currents

Topsy the Elephant (circa 1875 - January 4, 1903) was a member of a domesticated herd at Coney Island's Luna Park. She had been a part of the Forepaugh Circus. Topsy was deemed an ill-tempered and dangerous animal since she had killed three men in as many years, including an abusive trainer who tried to feed her a lit cigarette.

Because Topsy was so violent, her owners decided to put her to death. A proposal of hanging was abandoned after the American Society for the Prevention of Cruelty to Animals protested. The elephant was offered a carrot poisoned with cyanide, but did not eat it. Later, Thomas Edison suggested electrocution, using the Westinghouse alternating current system of electricity transmission, which Edison, a backer of direct current, argued was more dangerous than DC. The ASPCA found this suggestion acceptable, viewing electrocution as a more humane form of killing. Electrocution killed Topsy quickly. Edison recorded the execution with a motion picture camera, and showed his film to audiences around the country as part of his unsuccessful attempt to discredit AC.

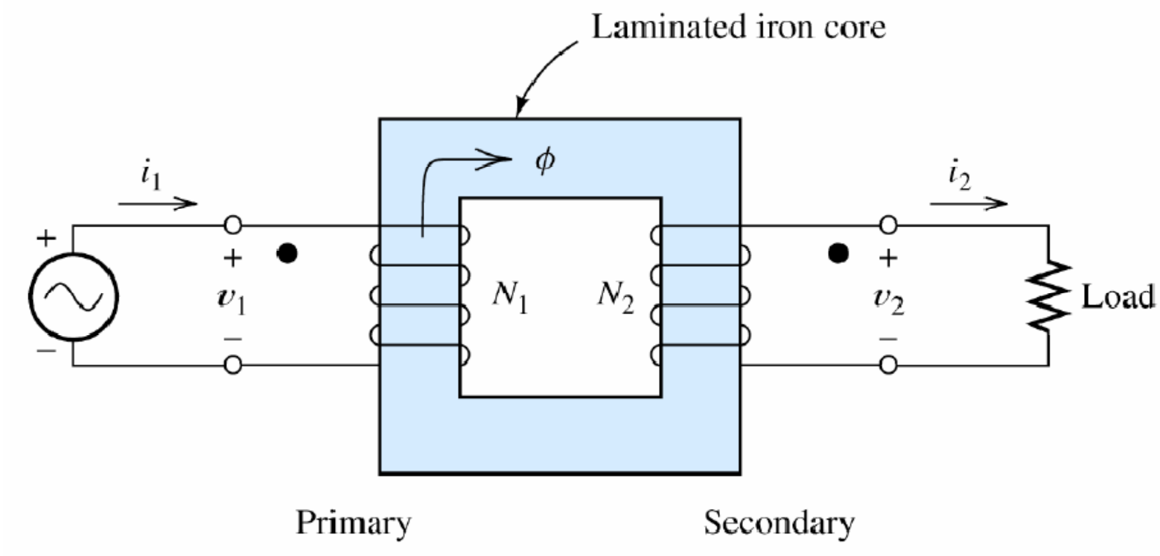
Ideal Transformers



$$v_1(t) = V_{1m}\cos(\omega t) = N_1 \frac{d\phi}{dt} \quad \text{by Faraday's Law}$$

$$\varphi(t) = \frac{1}{N_1}\int_0^t v_1(t)dt + \phi_0 = \frac{V_{1m}}{N_1}\int_0^t \cos(\omega t)dt = \frac{V_{1m}}{N_1\omega}\sin(\omega t)$$

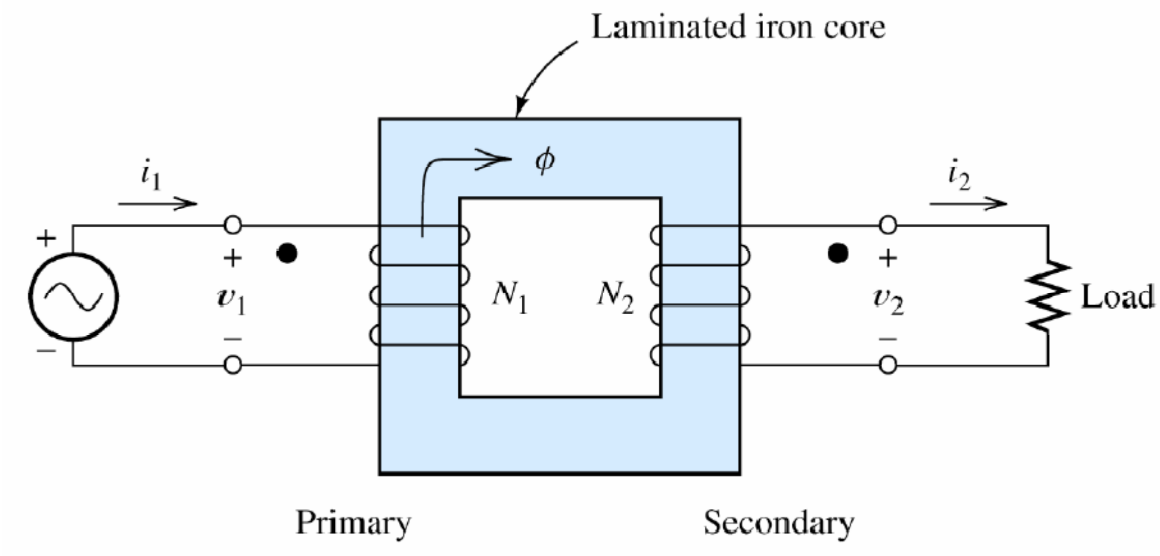
Ideal Transformers



$$v_2(t) = N_2 \frac{d\phi}{dt} = N_2 \frac{V_{1m}}{N_1 \omega} \frac{d}{dt}\left[\sin(\omega t)\right]$$

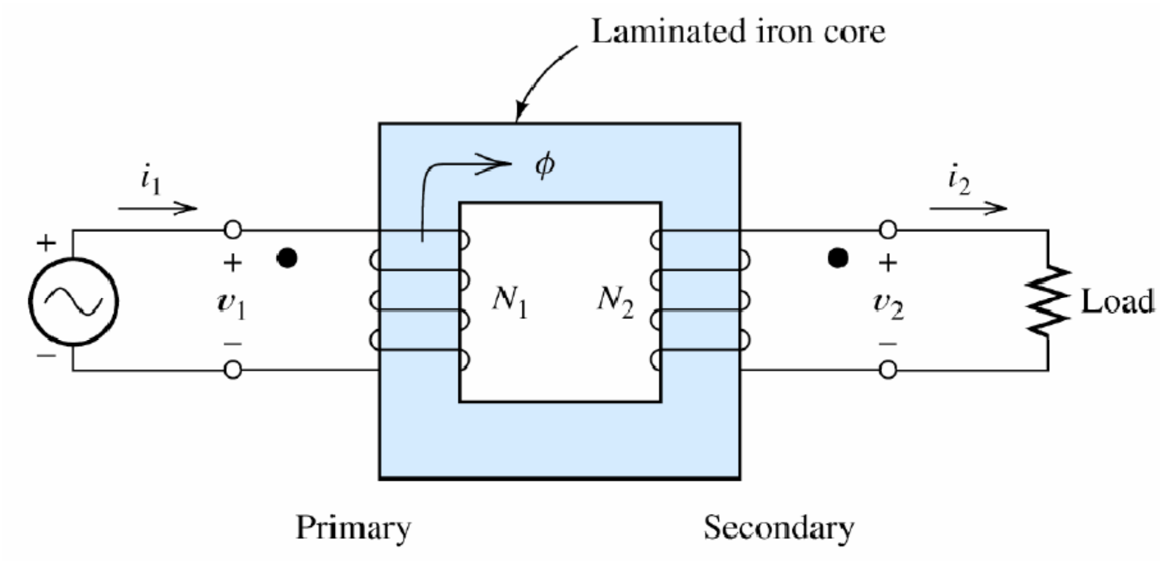
$$= \frac{N_2}{N_1} V_{1m} \cos(\omega t) = \frac{N_2}{N_1} v_1(t)$$

Ideal Transformers



$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

Ideal Transformers



The magneto-motive force (mmf) applied to the core:

$$\mathcal{F} = N_1 i_1 - N_2 i_2 = \mathcal{R}\phi \approx 0 \text{ since } \mathcal{R} \approx 0$$

$$N_1 i_1 = N_2 i_2$$

$$i_2(t) = \frac{N_1}{N_2} i_1(t)$$

If the voltage is stepped *up* the current is stepped *down*

Ideal Transformers

$$p_2(t) = v_2(t)i_2(t) = \frac{N_2}{N_1}v_1(t)\frac{N_1}{N_2}i_1(t)$$

$$= v_1(t)i_1(t) = p_1(t)$$

$$p_2(t) = p_1(t)$$

Net power is neither generated nor consumed by an *ideal* transformer

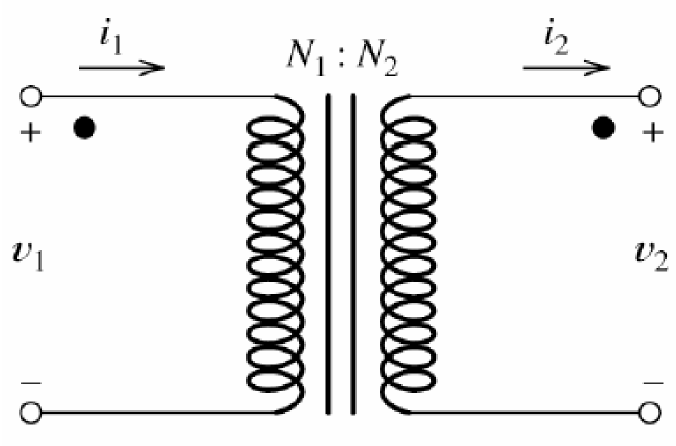
Ideal Transformers

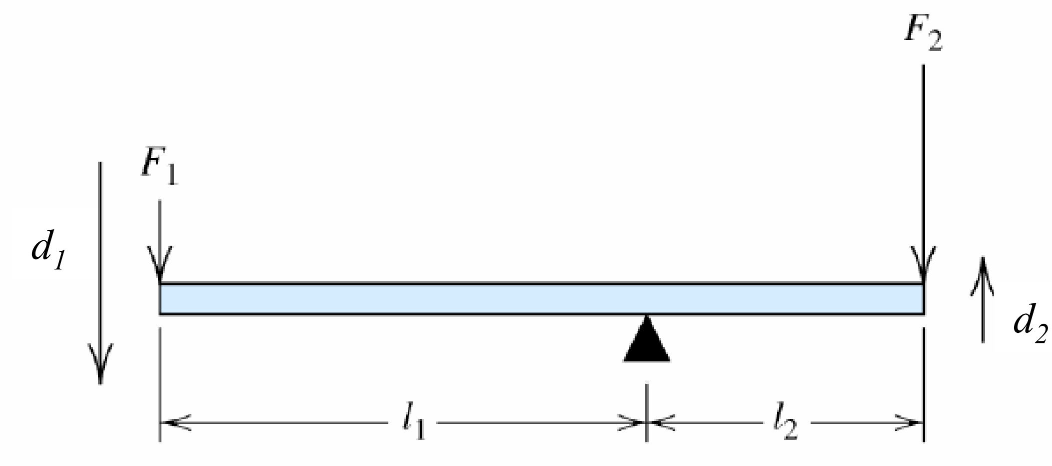
$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

$$i_2(t) = \frac{N_1}{N_2} i_1(t)$$

$$p_2(t) = p_1(t)$$

Mechanical Analog





$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

$$d_2 = \frac{l_2}{l_1} d_1$$

$$i_2(t) = \frac{N_1}{N_2} i_1(t)$$

$$F_2 = \frac{l_1}{l_2} F_1$$

Transformer Summary

1. We assumed that all of the flux links all of the windings of both coils and that the resistance of the coils is zero. Thus, the voltage across each coil is proportional to the number of turns on the coil.

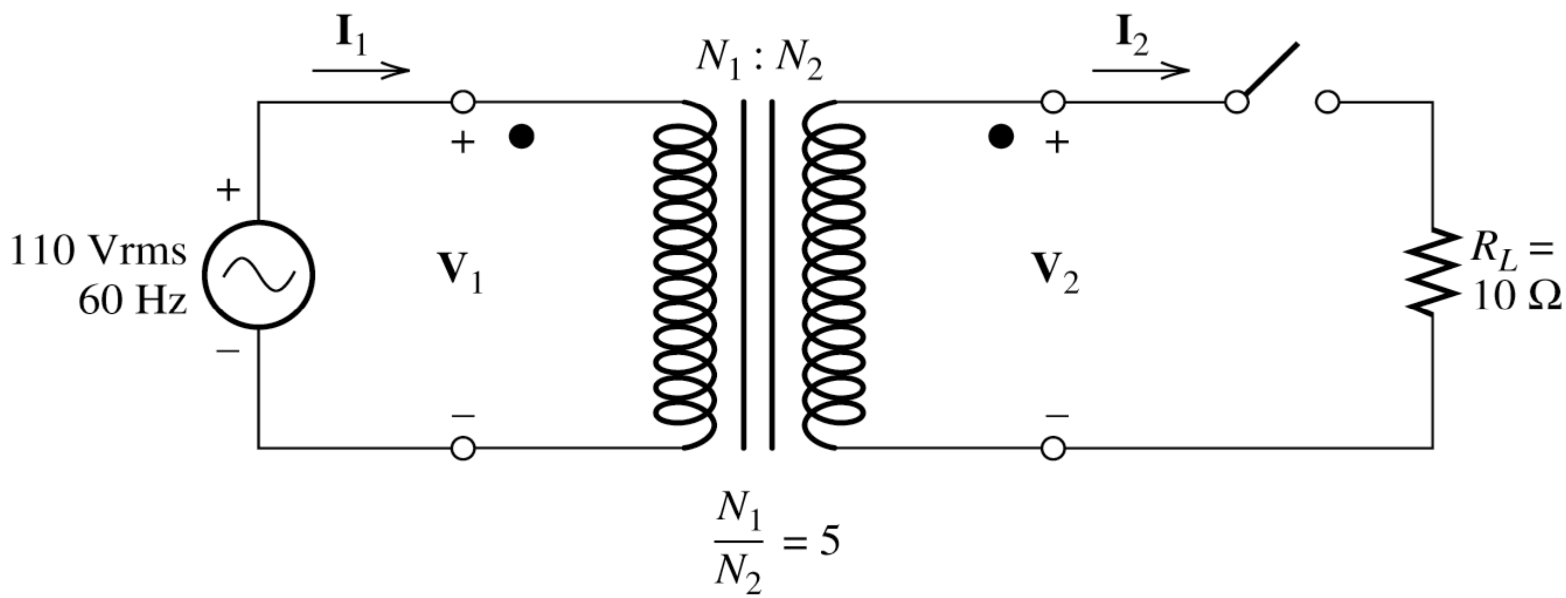
$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

2. We assumed that the reluctance of the core is negligible, so the total mmf of both coils is zero.

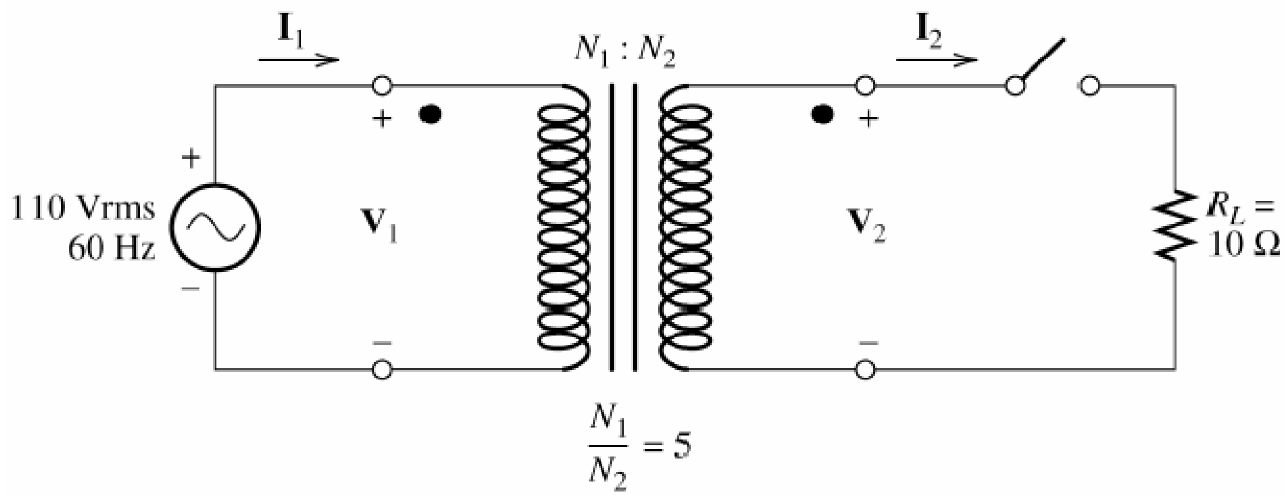
$$i_2(t) = \frac{N_1}{N_2} i_2(t)$$

3. A consequence of the voltage and current relationships is that all of the power delivered to an ideal transformer by the source is transferred to the load.

Analysis of a Circuit Containing an Ideal Transformer



Example

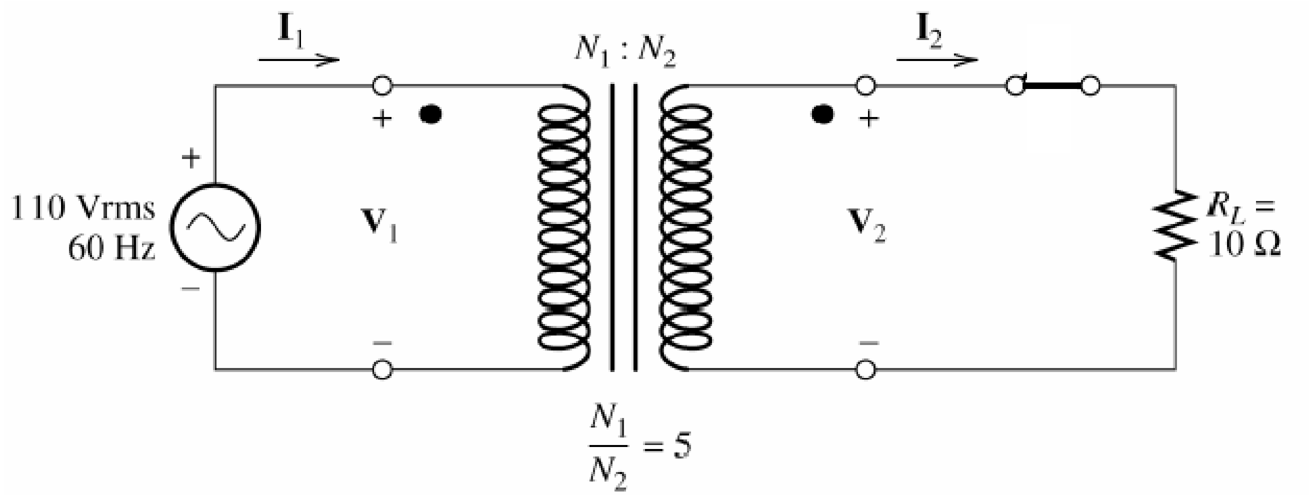


Find the rms values of the currents and voltages with the switch open and closed. With the switch open:

$$V_{2_{rms}} = \frac{N_2}{N_1} V_{1_{rms}} = \frac{1}{5} 110\, V_{rms} = 22\, V_{rms}$$

$$I_{1_{rms}} = \frac{N_2}{N_1} I_{2_{rms}} = 0 \rightarrow \text{No power is taken from the source}$$

Example

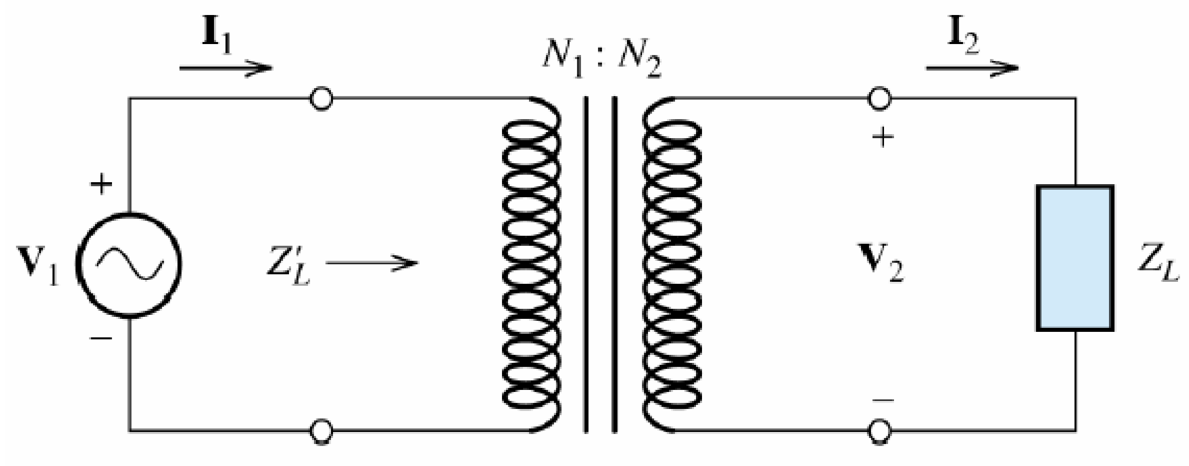


With the switch closed:

$$I_{2_{rms}} = \frac{V_{2_{rms}}}{R_L} = \frac{22V}{10\Omega} = 2.2A$$

$$I_{1_{rms}} = \frac{N_2}{N_1} I_{2_{rms}} = \frac{1}{5} \times 2.2A = 0.44A$$

Impedance Transformations

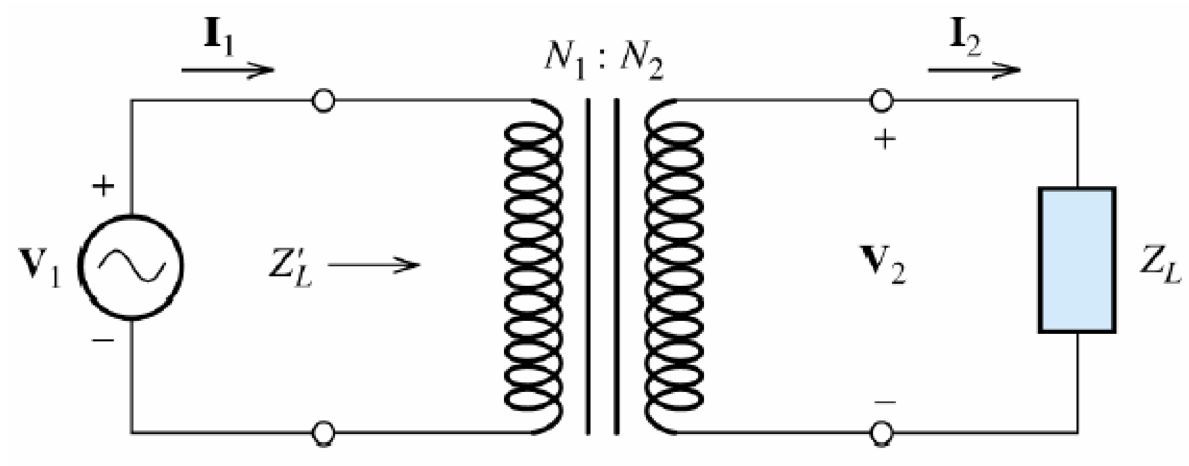


$$\frac{\mathbf{V}_2}{\mathbf{I}_2} = Z_L$$

$$= \frac{(N_2 / N_1)\mathbf{V}_1}{(N_1 / N_2)\mathbf{I}_1}$$

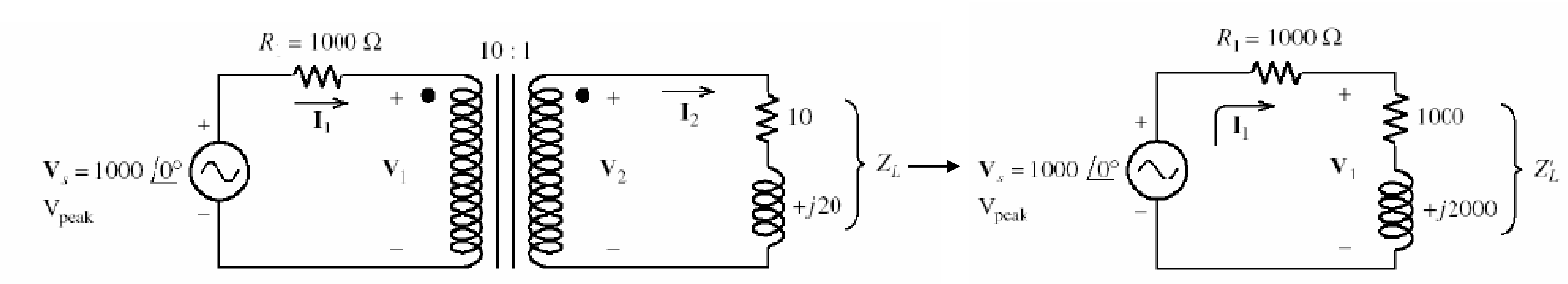
$$= \frac{(N_2 / N_1)}{(N_1 / N_2)} Z_L^{'} = \left(\frac{N_2}{N_1}\right)^2 Z_L^{'} \rightarrow Z_L^{'} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

Impedance Transformations



$$Z'_L = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

Example 15.11



Find the phasor voltages and currents. First, transform Z_L to the primary side of the transformer:

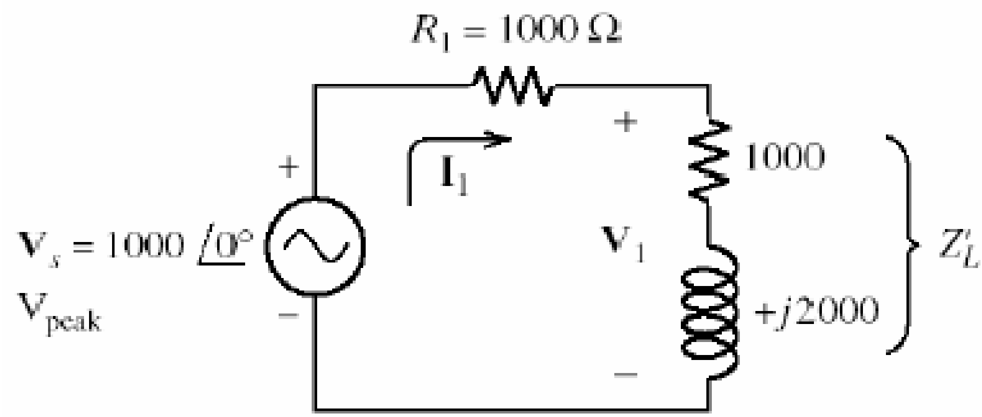
$$Z_L = 10 + j20$$

$$Z_{L'} = \left(\frac{N_1}{N_2}\right)^2 Z_L = (10)^2(10 + j20) = 1000 + j2000$$

$$Z_s = R_1 + Z_{L'} = 1000 + 1000 + j2000$$

$$= 2000 + j2000 = 2000\sqrt{2}\angle 45 = 2828\angle 45$$

Example 15.11



$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_s} = \frac{1000\angle 0^\circ}{2828\angle 45^\circ} = 0.3536\angle -45^\circ\ A$$

$$\mathbf{V}_1 = \mathbf{I}_1 Z_{L'} = (0.3536\angle -45^\circ)(1000 + j2000)$$

$$= (0.3536\angle -45^\circ)(2236\angle 63.43^\circ)$$

$$= 790.6\angle 18.43^\circ$$

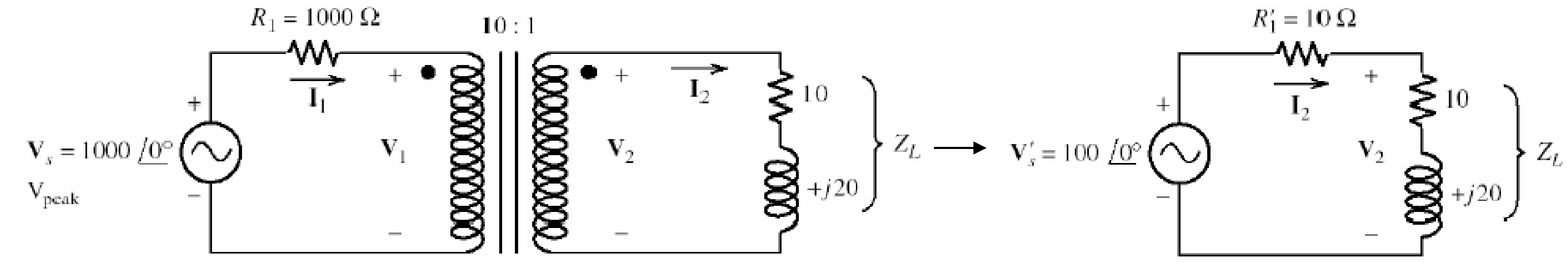
Example 15.11

We can now calculate the current and voltage phasors on the secondary side using the turns ratio:

$$\mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1 = 10(0.3536\angle -45^\circ) = 3.536\angle -45^\circ$$

$$\mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1 = \frac{1}{10}(790.6\angle 18.43^\circ) = 79.60\angle 18.43^\circ$$

Example 15.12

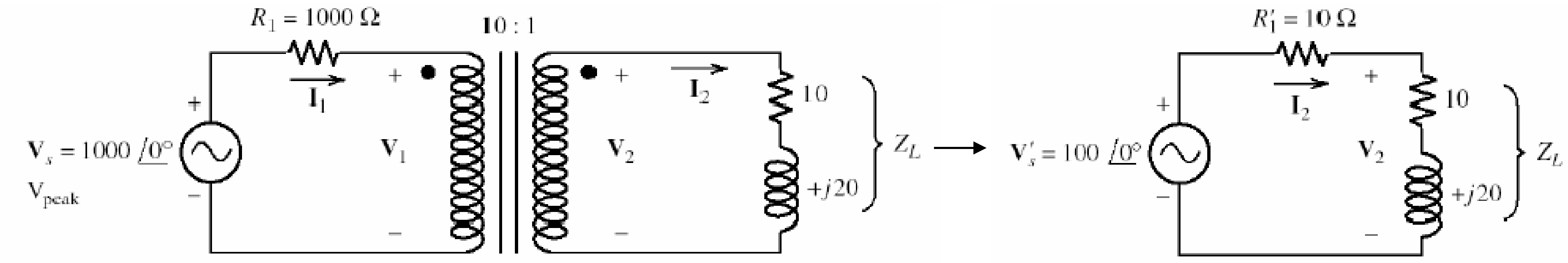


The voltage source and R_1 can also be transformed to the secondary side:

$$V_s^{'} = \frac{N_2}{N_1} V_s = \frac{1}{10} 1000\angle 0^\circ$$

$$R_1^{'} = \left(\frac{N_2}{N_1}\right)^2 R_1 = \left(\frac{1}{10}\right)^2 (1000) = 10\Omega$$

Example 15.12

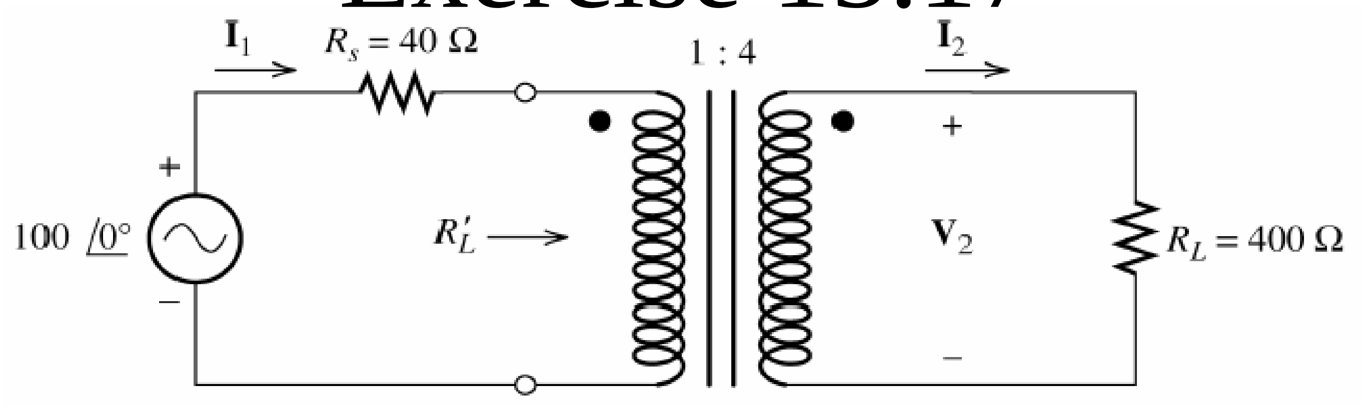


$$V_2 = \frac{10+j20}{10+10+j20}V_s^{'} = \frac{10+j20}{20+j20}(100\angle 0^\circ)$$

$$= \frac{\sqrt{100+400}\angle Tan^{-1}(20/10)}{20\sqrt{2}\angle 45^\circ}(100\angle 0^\circ)$$

$$= \frac{10\sqrt{5}\angle 63.43^\circ}{20\sqrt{2}\angle 45^\circ}(100\angle 0^\circ) = 79.06\angle 18.43^\circ$$

Exercise 15.17

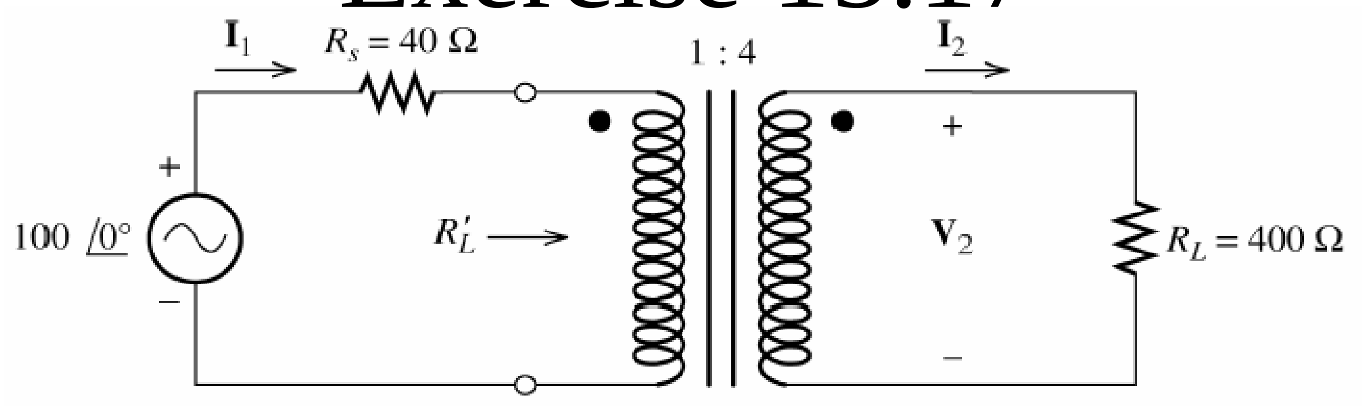


Find the values of $\mathbf{V}_2$ and the power delivered to the load. To find $\mathbf{I}_2$, transform $\mathbf{V}_s$ and R_s from the primary to the secondary side:

$$\mathbf{V}_s^{'} = \frac{N_2}{N_1}\mathbf{V}_s = \frac{4}{1}(100\angle 0^\circ) = 400\angle 0^\circ$$

$$R_S^{'} = \left(\frac{N_2}{N_1}\right)^2 R_S = 16R_S = 640\Omega$$

Exercise 15.17

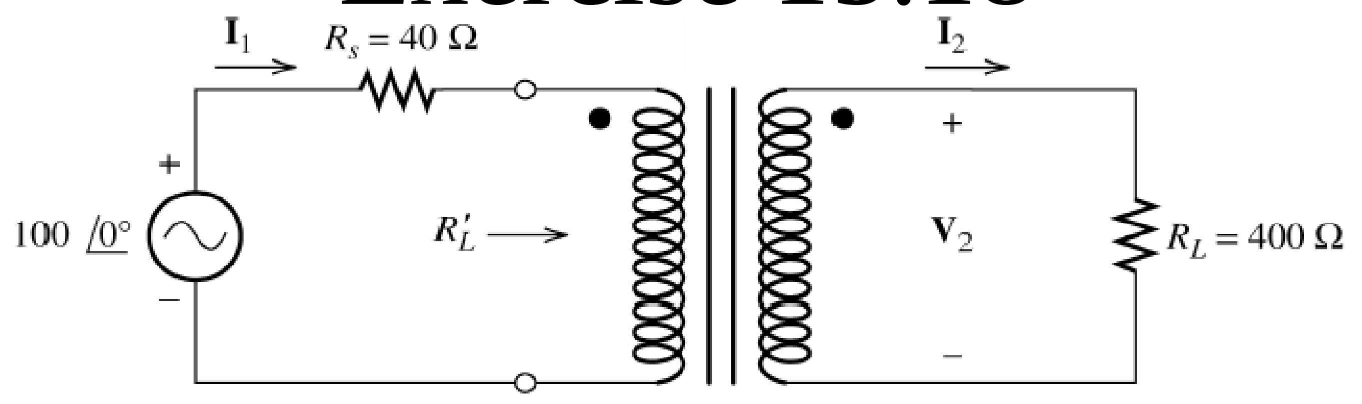


$$\mathbf{I}_2 = \frac{V_s'}{R_S' + 400\Omega} = \frac{400\angle 0^\circ}{640 + 400} = 0.385\angle 0^\circ$$

$$\mathbf{V}_2 = \mathbf{I}_2 R_L = (0.385\angle 0^\circ)(400\Omega) = 153.8\angle 0^\circ$$

$$P = (I_{2_{rms}})^2 R_L = \left(\frac{0.385}{\sqrt{2}}\right)^2 (400\Omega) = 29.6W$$

Exercise 15.18



Find the turns ratio that maximizes the power transfer to the load. What turns ratio transforms the 400Ω load resistance into a resistance of 40Ω on the primary side?

$$R_L^{'} = \left(\frac{N_1}{N_2}\right)^2 R_L = \left(\frac{N_1}{N_2}\right)^2 (400\Omega) = 40\Omega$$

$$\left(\frac{N_1}{N_2}\right)^2 = \frac{40\Omega}{400\Omega} = \frac{1}{10} \rightarrow \frac{N_1}{N_2} = \frac{1}{\sqrt{10}}$$

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